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Regular S-brane backgrounds

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Abstract: We construct time-dependent S-brane solutions to the supergravity field equations in various dimensions which (unlike most such geometries) do not contain curvature singularities. The configurations we consider are less symmetric than are earlier solutions, with our simplest solution being obtained by a simple analytical continuation of the Kerr geometry. We discuss in detail the global structure and properties of this background. We then generalize it to higher dimensions and to include more complicated field configurations — like non vanishing scalars and antisymmetric tensor gauge potentials — by the usual artifice of applying duality symmetries.

Keywords: Superstrings and Heterotic Strings, p-branes.

Contents

1. Introduction and summary

1.1 Introduction

Over the past years, considerable effort has been devoted to understanding the issue of time dependence in string theory. A practical place to begin the search for time-dependent string configurations is to find solutions to the field equations which describe the low-energy limit of string theory. The search for such solutions has received special attention, and has led to the study of the so called space-like branes or S-brane solutions [1].[1] Besides providing time-dependent backgrounds which have a string pedigree, the S-branes — embedded in asymptotically flat, time-dependent backgrounds — are believed to be relevant for describing the dynamics of tachyon fields [3].

Time-dependent configurations have an obvious appeal as potentially having cosmological applications [4, 5]. In particular, because the induced metric which observers embedded within these space-times can experience do not satisfy the usual Friedmann equation, they can undergo accelerated expansion, or bouncing [6], without necessarily requiring the

[1] See [2] for early work on this subject.

problematic kinds of matter which would normally be required. Since these spacetimes are time-dependent, they provide laboratories for studying particle production, such as has been considered in [7].

A large class of time dependent, asymptotically-flat S-brane solutions have been discussed in the literature [8, 9]. A common feature which these space-times typically share is that they are plagued by singularities. These can arise as null singularities, or naked time-like singularities inside internal static regions. This observation has led to the development of a theorem, which states that such singularities are inevitable for a fairly broad class of metrics [10] (see also [11]).

Although it may ultimately be possible to resolve these singularities within string theory, in this paper we instead ask whether singularity-free configurations can be found purely within the low-energy limit. We show that they can, by starting with a more general ansatz for the metric; a similar observation has been recently made in [12, 13]. The resulting exact solutions are typically less symmetric than the solutions already in the literature, but reduce to them in the limit where the asymmetries in the solution go to zero.

1.2 Summary

Our presentation comes in two parts. In the first one, section 2, we examine the properties of the simplest non-trivial, non-singular, time-dependent solution. In Part II this solution is generalized in several directions.

A simple solution. In the first part, we obtain the simplest solution by performing a suitable analytical continuation to the Kerr solution, in the same way that the well-known four-dimensional S0-brane solution,

$$ds^2 = -\frac{dt^2}{(1-2\,m/t)} + \left(1 - \frac{2\,m}{t}\right) dr^2 + t^2 d\mathcal{H}_2^2\,, \tag{1.1}$$

can obtained by the same kind of continuation from the static, spherical Schwarzschild black hole [9]. Here $d\mathcal{H}_2^2$ is the metric of the transverse spatial sections, which have a hyperbolic symmetry. This solution has an asymptotically-flat time-dependent region which is separated by a Cauchy horizon from an internal, static region containing a naked time-like singularity.

Performing a similar continuation of the Kerr rotating black hole solution produces a time-dependent spacetime having less symmetry, but which is completely regular and without curvature singularities.[2] The metric we would obtain is:

$$\begin{aligned} ds^2 \;=&\; -\frac{\Sigma}{\Delta}\,dt^2 + \left(\frac{\Delta + a^2\sinh^2\theta}{\Sigma}\right) dr^2 + \Sigma\, d\theta^2 + \\ &+ \left[(t^2+a^2)^2 + a^2\sinh^2\theta\Delta\right]\frac{\sinh^2\theta}{\Sigma}\,d\phi^2 - \frac{4m\,a\,t}{\Sigma}\sinh^2\theta\, d\phi\, dr\,, \end{aligned} \tag{1.2}$$

where

$$\begin{aligned} \Sigma \;&= t^2 + a^2\cosh^2\theta\,, \\ \Delta \;&= t^2 - 2mt + a^2 = (t-m)^2 + a^2 - m^2\,. \end{aligned}$$

[2] The observation that angular momentum can cure singularities in string theory was noticed in [14].

The solution depends on the parameter a, an integration constant which is the analog of the angular momentum of the Kerr solution, and a parameter m, that in the black hole case corresponds to the mass. The nature of the solution depends crucially on the value of a and m.

As we will see, when $a = 0$, we recover the S-brane solution of eq. (1.1), together with its singularity. However, as soon as a is nonzero, but small, the solution that results is free of curvature singularities. It instead has a stationary region which contains closed time-like curves (CTC's). This region is separated by Cauchy horizons, from external time-dependent regions.

More interestingly, for large enough values for the parameter a the geometry is *completely* regular and causally well-behaved. It is defined for all values of the coordinates and it does not contain horizons or singularities. In particular, we show that the geodesics are well-behaved everywhere.

Generalizations. In the second part, section 3, we generalize this simple geometry to more complicated situations. We do so in several ways. One non trivial extension is to d dimensions higher than four, where the $(d-4)$ extra dimensions are characterized by a hyperbolic symmetry. We also find new solutions by T-dualising the simpler configuration we found in the first part, along the lines of [15]. Other solutions are generated using similar solution-generating techniques in higher dimensions, including two examples of configurations with non-trivial dilaton and antisymmetric form fields in five dimensions. The first of these examples is obtained by applying T-duality plus coordinate rotations [16] to a known five-dimensional vacuum spacetime, leading to regular configurations with non-trivial NS fields. The second example applies a Hassan-Sen transformation [17] to a known solution, leading to a nontrivial time-dependent solution for low-energy heterotic string theory with nonzero one- and two-form fields. For each configuration, we identify the regions of parameter space for which our time-dependent solutions are completely regular.

2. Kerr S-branes

In this section we construct and discuss in detail the properties of the simplest regular S-brane solution. This is obtained as an analytical continuation of the Kerr black hole in vacuum four-dimensional Einstein gravity. We begin with a review of the Kerr black hole, followed by the construction of its S-brane generalization — the S-Kerr solution — and a study of its properties.

2.1 The Kerr geometry

The four dimensional Kerr black hole is the axially-symmetric solution to the vacuum Einstein equations which describes the spacetime outside of a rotating black hole. In Boyer-Lindquist coordinates, the solution is given by

$$ds^2 = -dt^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + (r^2+a^2)\sin^2\theta d\phi^2 + \frac{2mr}{\Sigma}\left(a\sin^2\theta d\phi - dt\right)^2, \qquad (2.1)$$

where

$$\Sigma = r^2 + a^2 \cos^2\theta$$
$$\Delta = r^2 - 2mr + a^2 = (r-m)^2 + a^2 - m^2\,. \tag{2.2}$$

In these expressions, m represents the mass of the black hole and $a \equiv J/m$ parameterizes the angular momentum per unit mass. This metric has the following properties on which we shall draw in what follows (see, *e.g.*, ref. [18] for details):

- It is asymptotically flat, as an examination of the Riemann tensor shows. The variable ϕ parameterizes the direction of the axial symmetry, and can be compactified to lie in the interval $[0, 2\pi]$ without introducing spurious conical singularities anywhere.

- It has two isometries, corresponding to the shifts in the coordinates t and ϕ in Boyer-Lindquist coordinates. These correspond physically to time-translation invariance and axial symmetry. There is indeed a mixed term $g_{t\phi}$ indicating that the solution rotates.

- It has a curvature singularity where the invariant $R^{\mu\nu\lambda\sigma}R_{\mu\nu\lambda\sigma}$ diverges. This occurs when $\Sigma = 0$, or equivalently when $r = 0$ and $\theta = \pi/2$ simultaneously. This is the standard ring singularity of the Kerr geometry.

- It has coordinate singularities when $\Delta = 0$, which turn out to be horizons of the Kerr geometry. When $a^2 < m^2$ the metric has two horizons.[3] The internal one hides a ring singularity located at $\Sigma = 0$, that is for $r = 0$ and $\theta = \pi/2$. There is also an ergoregion which is determined by the zeroes of g_{tt}, where the infinite red shift surfaces are located. The outer boundary of the ergosphere is located outside the outer horizon. On the other hand, when $a^2 > m^2$, the metric does not have horizons and the ring singularity is naked.

- The Kerr geometry has closed time-like curves, for small negative values of r. When $a^2 < m^2$, these are safely limited to a stationary region inside the event horizon.

These properties change dramatically for the Kerr S-brane, as we now show.

2.2 The Kerr S-brane

In order to construct the simplest Kerr S-brane solution we follow the steps taken to obtain the more symmetric S-brane solutions from the simpler Schwarzschild geometry. This is done via a suitable analytical continuation of the Schwarzschild black hole introduced in [7, 9], leading to a spacetime with time-like naked singularities. In the same spirit we perform the following analytic i-rotation [9] in the Kerr geometry:

$$t \to ir\,, \qquad r \to it\,, \qquad \theta \to i\theta\,, \qquad a \to ia\,, \qquad m \to im\,. \tag{2.3}$$

[3]Cosmological aspects of the time dependent region inside the two horizons has been discussed in [19].

This leads to the following time-dependent, axially symmetric solution to the four dimensional Einstein equations in vacuum:

$$
\begin{aligned}
ds^2 &= -\frac{\Sigma}{\Delta}\, dt^2 + \left(\frac{\Delta + a^2 \sinh^2\theta}{\Sigma}\right) dr^2 + \Sigma\, d\theta^2 + \\
&\quad + \left[(t^2+a^2)^2 + a^2 \sinh^2\theta \Delta\right] \frac{\sinh^2\theta}{\Sigma}\, d\phi^2 - \frac{4m\,a\,t}{\Sigma} \sinh^2\theta\, d\phi\, dr\,,
\end{aligned}
\tag{2.4}
$$

where now

$$
\begin{aligned}
\Sigma &= t^2 + a^2 \cosh^2\theta\,, \\
\Delta &= t^2 - 2mt + a^2 = (t-m)^2 + a^2 - m^2\,.
\end{aligned}
\tag{2.5}
$$

The solution (2.4) inherits two isometries from the Kerr solution, corresponding to shifts in the coordinates r and ϕ. The mixed term $g_{t\phi}$, indicating the rotational structure of the Kerr black hole, has now been replaced by a mixed term $g_{r\phi}$, indicating a "rotation" in space, or, better, a helical or wrung structure. The solution is also invariant under the exchange $a \to -a$ and $m \to -m$, a transformation whose physical interpretation we discuss in the next section. Notice too that we recover the well-known Schwarzschild i-rotated solution or S0-brane (complete with its time-like naked singularities at the origin) in the limit $a \to 0$.

The key property of this new solution follows from the observation that, unlike for the Kerr geometry, the quantity Σ cannot be zero if $a \neq 0$. As a consequence there is no analog of the Kerr ring singularity for this new solution. For instance, the curvature invariant $R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$ is given by the following expression:

$$
\begin{aligned}
R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} &= \frac{720m^2}{(t^2+a^2\cosh\theta^2)^9}\left(t^2 - 2mt + a^2\cosh\theta^2\right) \times \\
&\quad \times \left[\left(t^4 - 6a^2t^2\cosh\theta^2 + a^4\cosh\theta^4\right)^2 - \left(4at^3\cosh\theta + 4a^3 t\cosh\theta\right)^2\right],
\end{aligned}
\tag{2.6}
$$

from which it is clear that this invariant is well defined everywhere so long as $a \neq 0$.[4]

2.3 Global structure

We now examine the global structure of the Kerr S-brane in more detail.

Asymptopia. As might be expected, the Kerr S-brane inherits an asymptotically flat region from the Kerr black hole. This property is less evident in the Boyer-Lindquist form of the metric used above due to the presence of the diverging sinh and cosh functions. To display this region more explicitly it is convenient instead to pass to Kerr-Schild coordinates, in terms of which the solution may be rewritten as

$$
ds^2 = d\bar{r}^2 + dx^2 + dy^2 - dz^2 - \frac{2mt^3}{t^4+a^2z^2}\left(\frac{t(xdx+ydy) - a(xdy-ydx)}{t^2+a^2} + \frac{zdz}{t} + d\bar{r}\right)^2,
\tag{2.7}
$$

[4]In [20], a similar procedure of analytic continuation has been used to obtain regular time-dependent solutions in the presence of a cosmological constant.

where

$$x + iy = (t + ia) \sinh\theta \exp\left[i \int (d\phi + a\Delta^{-1} dt)\right]$$
$$z = t \cosh\theta\,, \qquad \bar{r} = \int (dr + (t^2 + a^2)\Delta^{-1} dt) - t\,. \tag{2.8}$$

Here t is to be regarded as being a function of the other coordinates, as determined implicitly from the condition

$$t^4 - (z^2 - x^2 - y^2 - a^2)t^2 - a^2 z^2 = 0\,. \tag{2.9}$$

From these expressions it is easy to see that for large values of the coordinates the geometry approaches the flat space, since in this limit the second line of eq. (2.7) vanishes.

Location of horizons. Although we have seen that the metric does not have curvature singularities, in its form (2.4) it does have coordinate singularities for those points where the function Δ vanishes, which happens when

$$t = t^h_\pm \equiv m \pm \sqrt{m^2 - a^2}\,. \tag{2.10}$$

These represent horizons of the geometry. From eq. (2.10) we see that the global properties of the geometry depend crucially on the relative size of the parameters a and m, and so we now discuss the various cases separately.

2.3.1 The case $a^2 < m^2$: closed timelike curves

In this case the configuration has two horizons, which separate two asymptotically-flat and time-dependent regions from a stationary region which contains closed timelike curves. This situation is illustrated in figure 1. If $a^2 = m^2$ then the two horizons degenerate

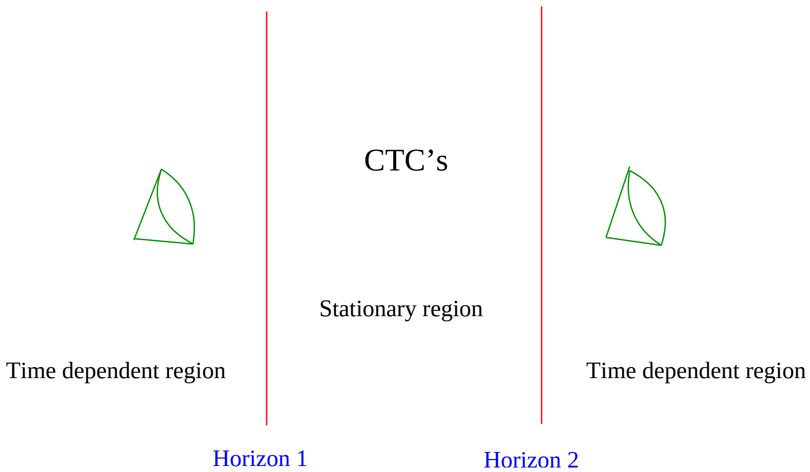

Figure 1: Pictorial representation of the Kerr S-brane solution (2.4), when $a^2 < m^2$. In the external, time dependent regions, we draw examples of future light cones. Notice that the internal, stationary region contains CTC's, making it meaningless to represent future light cones.

into one. Notice that the two external time-dependent regions get interchanged with one another under the transformation $a \to -a$ and $m \to -m$, which leaves the metric invariant.

The absence of a time-like curvature singularity for $0 < a^2 < m^2$, with its replacement by a second horizon, allows a new interpretation of the time-like singularity which appears in the limiting case $a = 0$. It suggests that the time-like singularity, found in this limit might be thought of as a second S-brane, (that is, an asymptotically-flat and time-dependent region bounded by a horizon), but in the limit where its horizon collapses to zero size and becomes singular. If a is small but nonzero, then the singular horizon becomes regular and the geometry describes two S-branes separated by the internal stationary region. In this case, there is also an equivalent of the Kerr's ergoregion, limited by the boundaries located at the points where $\Delta + a^2 \sinh\theta = 0$. The outer of these surfaces is located inside the first horizon, in contrast with the Kerr black hole case.

Although there are no curvature singularities, the stationary interior region of the geometry for $a^2 < m^2$ contains closed timelike curves, and this means that it is not causally well defined. The existence of these curves arises because in the stationary region it is the coordinate ϕ which becomes time-like,[5] since the coefficient of $d\phi^2$ in (2.4) becomes negative:

$$\left[(t^2 + a^2)^2 + a^2 \sinh^2(\theta)\Delta\right] \leq 0\,. \tag{2.11}$$

This condition can be satisfied whenever $\Delta < 0$ — that is, *only* in the stationary part of the solution, inside the Cauchy horizons. Nevertheless, since there are no event horizons hiding these causally-problematic regions from external observers, it is not clear that the geometry in the case $a^2 < m^2$ has a sensible physical interpretation [22].[6]

2.3.2 The Case $a^2 > m^2$: Nonsingular Geometries

When $a^2 > m^2$ neither Σ nor Δ can vanish, and so the geometry we obtain is well defined and time-dependent everywhere, and asymptotically approaches flat space. So long as $m \neq 0$ the geometry is curved, because its Riemann tensor does not vanish.[7] The resulting configuration is time dependent and does not contain horizons or curvature or metric singularities. As we now show, the geodesics for this geometry are perfectly well defined. The Penrose diagrams along the symmetry axis (that is, for $\theta = 0$) for all the solutions are presented in figure 2.

Geodesic motion. Let us now consider the geodesics of the time dependent geometry for $a^2 > m^2$. We restrict ourselves here to the simplest case for which the coordinate θ does not vary.

[5]A simple closed timelike curve is then obtained by moving along trajectories on which only ϕ varies, given that ϕ is a periodic variable with period 2π. However, notice that the presence of CTC's does *not* rely on the periodicity of ϕ (see [21] for more details).

[6]As we will see in the second part of this letter, also adding other fields motivated by string theory, we find regular solutions with CTC's. Although it is not yet clear whether a chronology protection agency exists in string theory, there are some indications that this might be indeed the case (see e.g. [23]).

[7]When $m = 0$ the geometry is flat, being Minkowski space-time in an unusual coordinate system (as is most apparent by choosing $m = 0$ in formula (2.7)).

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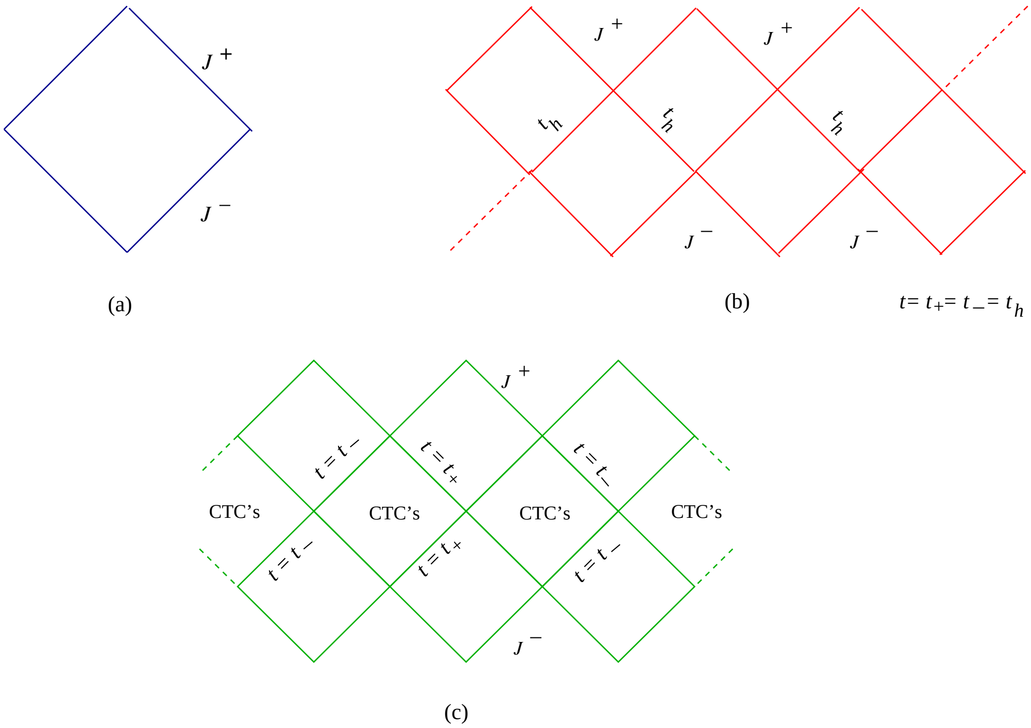

Figure 2: Penrose diagrams for the S-Kerr brane for (a) $a^2 > m^2$, (b) $a^2 = m^2$, (c) $a^2 < m^2$.

Due to the two isometries there are two conserved quantities which provide two first-integrals of the geodesic equations. These can be defined by

$$\begin{aligned} -P &= \left(\frac{2mt}{\Sigma} - 1\right)\dot{r} + \frac{2mt}{\Sigma} a \sinh^2\theta \dot{\phi}\,, \\ L &= -\frac{2mt}{\Sigma} a \sinh^2\theta\,\dot{r} + \left[(t^2+a^2)^2 + a^2\sinh^2\theta\Delta\right]\frac{\sinh^2\theta}{\Sigma}\,\dot{\phi}\,. \end{aligned} \tag{2.12}$$

Where a dot means derivative with respect to the geodesic affine parameter. In terms of these the geodesic equations can be written as follows:

$$\epsilon = -\frac{\Sigma}{\Delta}\dot{t}^2 + \frac{\Delta}{\Sigma}\left(\dot{r} + a\sinh^2\theta\dot{\phi}\right)^2 - \frac{\sinh^2\theta}{\Sigma}\left[(t^2+a^2)\dot{\phi} - a\dot{r}\right]^2, \tag{2.13}$$

$$\begin{aligned} 0 = &-\frac{a^2}{\Delta}\dot{t}^2 - \frac{a^2\Delta}{\Sigma^2}\left(\dot{r} + a\sinh^2\theta\,\dot{\phi}\right)^2 + \frac{2a\Delta\dot{\phi}}{\Sigma}\left(\dot{r} - a\sinh^2\theta\,\dot{\phi}\right) + \\ &+\frac{(t^2+a^2)}{\Sigma^2}\left[(t^2+a^2)\dot{\phi} - a\dot{r}\right]^2, \end{aligned} \tag{2.14}$$

where $\epsilon = 0\,(-1,+1)$ for null (time-like, space-like) geodesics. Simple algebraic manipulations reveal that, starting from the previous equations characterizing the geodesics, the invariants P and L satisfy the following relation:

$$(L - aP\sinh^2\theta)(L + aP\sinh^2\theta) = -\epsilon\, a^2\sinh^4\theta\,. \tag{2.15}$$

We now discuss the behavior of null and time-like geodesics in more detail.

Null Geodesics. In this case, the relation (2.15) considerably simplifies the equations of motion. Using the condition

$$(L - aP \sinh\theta^2) = 0\,, \tag{2.16}$$

the geodesic equations simplify to the following expressions for the first derivatives along the geodesic affine parameter:

$$\dot{t} = \pm P\,, \qquad \dot{r} = \frac{(t^2 + a^2)}{\Delta} P \qquad \text{and} \qquad \dot{\phi} = \frac{aP}{\Delta}\,. \tag{2.17}$$

These first-order differential equations are well-defined for any value of the coordinates and of the affine parameter. Because Δ cannot vanish, these equations can be integrated without introducing singularities, leading to geodesics which are also everywhere well-defined, making the space-time geodesically complete for null geodesics.

Time-like geodesics. The equations for timelike geodesics are harder to handle. We concentrate on the simplest situation, for which the constant coordinate θ is chosen to vanish: $\theta = 0$. We obtain in this case

$$\begin{aligned}
\dot{t} &= \pm\left(P^2 + \frac{\Delta}{\Sigma}\right), \\
\dot{r} &= \frac{\Sigma}{\Delta} P\,, \\
\dot{\phi} &= \frac{aP}{\Sigma}\left(\frac{\Sigma}{\Delta} - 1\right)\left[1 \pm \sqrt{1 + \frac{\Delta^2\Sigma}{P^2(\Delta-\Sigma)^2}\left[\frac{P^2}{\Delta}\left(1 + \frac{1}{a}\right) + \frac{1}{\Sigma} - \frac{P^2\Sigma}{\Delta}\right]}\,\right].
\end{aligned} \tag{2.18}$$

It is also true in this case that the geodesics which result from these equations are well-defined for all values of the coordinates, and the space time is geodesically complete for these time-like geodesics.

3. Generalizations

In this section we generalize in various ways the time dependent Kerr S-brane solution of the last section. We first extend the four dimensional pure-gravity solution to a more general solution of the combined Einstein-dilaton-antisymmetric tensor equations of 4D supergravity, for which the dilaton and antisymmetric tensor fields do not vanish. This new solution is obtained by performing a T-duality transformation, exploiting the T-duality invariance of the supergravity action.

Next we extend the solutions in the vacuum to higher dimensions: we obtain these solutions by performing a complex substitution in stationary solutions already known in the literature.

We then continue with two examples, which show how to generalize higher-dimensional solutions by acting on them with symmetries. Similar to the 4D case after T-duality, the new time-dependent solutions obtained in this way contain nontrivial dilaton, and are coupled to two-forms and/or one forms.

We obtain the first of these higher-dimensional examples by performing a coordinate rotation, followed by a proper T-duality transformation, leading to a 5-dimensional S-string configuration, involving a background dilaton field and an antisymmetric three form field. In the second example, we start with an S-string in five dimensional vacuum, and, by means of applying a Hassan-Sen transformation, we extend it to a solution within a more general background, containing in addition nonzero abelian gauge fields. Their global properties are richer than the four dimensional ones, and we discuss them in some detail.

3.1 Four dimensional T-dual configurations

Although our Kerr S-brane configuration so far has been considered as a solution to the vacuum Einstein equations, it may also be regarded as a solution to the combined Einstein-Dilaton-Antisymmetric tensor field equations of supergravity models, for which the dilaton, Φ, is a constant and the antisymmetric tensor field, $B_{\mu\nu}$, vanishes. The action for this model, in the string frame, is given explicitly by

$$S = -\int dx^4 \sqrt{-g_4}\, e^{-\Phi} \left[R + (\nabla\Phi)^2 - \frac{1}{12} H_3^2 \right], \tag{3.1}$$

where the 3-form H_3 is related to the 2-form potential, B_2, by $H_3 = \mathrm{d}B_2 + \ldots$, where the dots denote possible Chern-Simons terms which play no role in what follows.

Given any solution to the field equations of this action which enjoy at least one continuous symmetry, there is a well-known prescription for generating new solutions. The new solution is obtained by performing a T-duality transformation based on one of the original solution's symmetries [24]. As applied to the translation symmetry, $r \to r + \text{constant}$, of the Kerr S-brane solution, the new solution which is generated in this way is given by the fields $\tilde{g}_{\mu\nu}$, $\tilde{B}_{\mu\nu}$ and $\tilde{\Phi}$, which are related to the Kerr S-brane solution (in the string frame) by the explicit transformation

$$\begin{aligned}
\tilde{g}_{rr} &= \frac{1}{g_{rr}}, \qquad \tilde{g}_{r\alpha} = \frac{B_{r\alpha}}{g_{rr}}, \qquad \tilde{g}_{\alpha\beta} = g_{\alpha\beta} - \frac{(g_{r\alpha}g_{r\beta} - B_{r\alpha}B_{r\beta})}{g_{rr}}, \\
\tilde{B}_{r\alpha} &= \frac{g_{r\alpha}}{g_{rr}}, \qquad \tilde{B}_{\alpha\beta} = B_{\alpha\beta} - \frac{2g_{r[\alpha}B_{\beta]r}}{g_{rr}}, \\
\tilde{\Phi} &= \Phi - \ln g_{rr},
\end{aligned} \tag{3.2}$$

where α and β denote all of the coordinate directions except for r.

This transformation furnishes us with the following new solution for the action (3.1), describing a time dependent configuration in the presence of nontrivial dilaton and antisymmetric tensor field:

$$\begin{aligned}
d\tilde{s}^2 &= -\frac{\Sigma}{\Delta}dt^2 + \frac{\Sigma}{\Delta + a^2\sinh^2\theta}dr^2 + \Sigma\, d\theta^2 + \frac{\Delta\Sigma\sinh^2\theta}{\Delta + a^2\sinh^2\theta}d\phi^2, \\
\tilde{\Phi} &= -\ln\left[\frac{\Delta + a^2\sinh^2\theta}{\Sigma}\right], \\
\tilde{B}_{\phi r} &= -\frac{2mat\sinh^2\theta}{\Delta + a^2\sinh^2\theta}.
\end{aligned} \tag{3.3}$$

This configuration is completely time-dependent, and regular, provided that $\Delta > 0$, that is $a^2 > m^2$, just as was the original solution before performing the T-duality. In this case, the parameter a, whose size rules the regularity of the solution, is associated with the "charge" of the non vanishing antisymmetric two form. The scalar field is also completely regular for this range of a, and it depends on both the coordinates t and θ. The dilaton configuration has a global maximum and a global minimum, and it asymptotically vanishes: it is represented in figure 3.

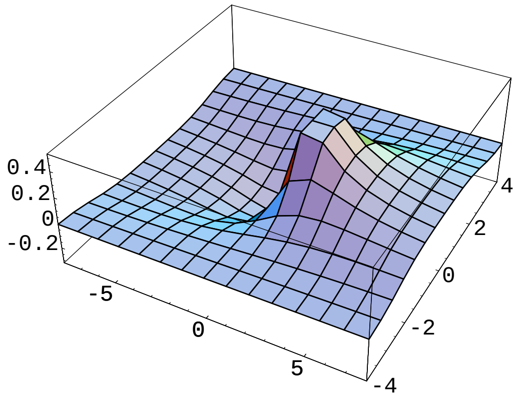

Figure 3: Behavior of the scalar field for the solution in eq. (3.3), for $m = 1$, $a = 1.2$.

For $a^2 < m^2$, the metric has Cauchy horizons at the surfaces $\Delta = 0$. The geometry is characterized by the presence of internal stationary regions, containing CTC's, and curvature singularities when the dilaton diverges, on the curves given by the solution of the equation

$$t^2 - 2mt + a^2 \cosh^2 \theta = 0 \,. \tag{3.4}$$

For this four dimensional solution with $a^2 < m^2$, in the presence of non trivial dilaton, it is not possible to find a configuration free of curvature singularities. We will show, in subsection 3.3, how this condition can be reached, in a higher dimensional example, switching on additional components of the antisymmetric tensor field.

3.2 Higher dimensional S-Kerr solutions

The vacuum solution in four dimensions that we discussed in the first part of this letter, can be generalized to d dimensional vacuum solutions, at least in two different ways. The first, immediate way, is to simply add to the solution $(d-4)$ flat space-like directions (we will use this observation in the next subsection as starting point to find non trivial configurations via solution generating techniques). More interestingly, we can add $(d-4)$ extra dimensions, characterized by a hyperbolic symmetry. This can be done by applying a suitable analytic continuation to the higher dimensional Kerr black hole with only one rotation parameter, which was found in ref. [25]. The stationary solution, starting point for our discussion, is given by

$$\begin{aligned} ds_d^2 = &-\frac{\Delta - a^2 \sin^2\theta}{\Sigma}\, dt^2 + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma\, d\theta^2 + \frac{4\, m\, a\, r^{5-d} \sin^2\theta}{\Sigma}\, dt\, d\phi + \\ &+ \frac{\sin^2\theta}{\Sigma}[(r^2+a^2)^2 - \Delta\, a^2 \sin^2\theta] d\phi^2 + r^2 \cos^2\theta\, d\Omega_{d-4}^2 \,, \end{aligned} \tag{3.5}$$

where

$$\begin{aligned} \Delta &= r^2 + a^2 - 2\, m\, r^{5-d} \\ \Sigma &= r^2 + a^2 \cos^2\theta \end{aligned} \tag{3.6}$$

JHEP04(2004)038

and $d\Omega_{d-4}$ is the line element of the $(d-4)$ sphere. The horizons are located at the points where $\Delta = 0$.

Consider now the following complex replacement

$$t \to ir\,, \qquad r \to it\,, \qquad \theta \to i\theta\,, \qquad a \to ia\,, \qquad m \to i^{d-3}m \qquad d\Omega_{d-4} \to id\mathcal{H}_{d-4} \tag{3.7}$$

where $d\Omega_p$ and $d\mathcal{H}_p$ respectively denote the line elements for the p-sphere and p-hyperbola. Applied to the solution eq. (3.5) this transformation leads to the following time-dependent solution in d dimensions:

$$\begin{aligned} ds_d^2 = & -\frac{\Sigma}{\Delta}\,dt^2 + \frac{\Delta + a^2\,\sinh^2\theta}{\Sigma}\,dr^2 + \Sigma\,d\theta^2 - \frac{4\,m\,a\,t^{5-d}\,\sinh^2\theta}{\Sigma}\,dr\,d\phi + \\ & + \frac{\sinh^2\theta}{\Sigma}[(t^2+a^2)^2 + \Delta\,a^2\,\sinh^2\theta]\,d\phi^2 + t^2\cosh^2\theta\,d\mathcal{H}_{d-4}^2\,, \end{aligned} \tag{3.8}$$

where now

$$\begin{aligned} \Delta &= t^2 + a^2 - 2\,m\,t^{5-d} \\ \Sigma &= t^2 + a^2\,\cosh^2\theta\,. \end{aligned} \tag{3.9}$$

This configuration has horizons at the locus of points defined by the solutions to the equation

$$\Delta = t^2 + a^2 - 2mt^{5-d} = 0 \qquad \text{or} \qquad t^{d-3} + a^2\,t^{d-5} - 2m = 0\,. \tag{3.10}$$

From this equation is easy to see when, and how many, horizons are present. One finds that, in five dimensions, the geometry is completely time dependent, without horizons or curvature singularities, for $a^2 > 2m$. When this last inequality is not satisfied, an internal, regular stationary region is present, which contains CTC's.

Starting from six dimensions, the global structure changes dramatically. Let us consider only the region for positive t, since there is a curvature singularity at the origin. Although the metric is asymptotically time dependent, for positive m it is characterized by the presence of a Cauchy horizon, that separates the time-dependent region from an internal, stationary region containing a naked singularity at the origin. For negative m, the time dependent geometry is characterized by an initial, space-like singularity, since there are no horizons present.

Consequently, for six and more dimensions, our analytic continuation applied to (3.5) furnishes a singular geometry with a global structure similar to previously known S-branes.

3.3 A nonsingular charged S-string

In this section we next construct a five dimensional configuration which represents a solution to the field equations for the following string-frame action in five dimensions:

$$S = -\int dx^5\sqrt{-g_5}\,e^{-\Phi}\left[R + (\nabla\Phi)^2 - \frac{1}{12}\,H_3^2\right]. \tag{3.11}$$

As before, Φ is the dilaton while H_3 the antisymmetric 3-form field strength: $H_3 = \mathrm{d}B_2$.

It is relatively easy to find new solutions for this system. In order to find them we proceed in an analogous way to that outlined in [16], using the following three steps.

Step 1: we start with the vacuum spacetime of section (2.2) — i.e. eq. (2.4) — and we add to it an extra flat direction,[8] whose coordinate we call x. The solution obtained in this way corresponds to a time-dependent S-string configuration, and is given by

$$\begin{aligned} ds^2 &= -\frac{\Sigma}{\Delta}dt^2 + \left(\frac{\Delta + a^2\sinh^2\theta}{\Sigma}\right)dr^2 + \Sigma d\theta^2 + \\ &\quad + \left[(t^2+a^2)^2 + a^2\sinh^2\theta\Delta\right]\frac{\sinh^2\theta}{\Sigma}d\phi^2 - \frac{4m\,a\,t}{\Sigma}\sinh^2\theta d\phi dr + dx^2 \\ \Phi &= 0 \\ B &= 0\,, \end{aligned} \tag{3.12}$$

where we use the same conventions as in subsection 2.2. This solution has the same global properties as that of the 4-dimensional SKerr solution.

Step 2: we rotate the solution in the (r, x) plane, by the transformation

$$r = \hat{r}\cos\alpha + \hat{x}\sin\alpha\,, \qquad x = \hat{x}\cos\alpha - \hat{r}\sin\alpha\,. \tag{3.13}$$

We obtain the following rotated solution

$$\begin{aligned} d\hat{s}_5^2 &= -\frac{\Sigma}{\Delta}\,dt^2 + \left[\frac{\Delta + a^2\sinh^2\theta + 2mt\sin^2\alpha}{\Sigma}\right]d\hat{r}^2 + \left[\frac{\Delta + a^2\sinh^2\theta + 2mt\cos^2\alpha}{\Sigma}\right]d\hat{x}^2 + \\ &\quad + \Sigma\,d\theta^2 + \frac{\sinh^2\theta}{\Sigma}\left[(t^2+a^2)^2 + a^2\,\sinh^2\theta\Delta\right]d\phi^2 - \frac{4\,m\,t\,\cos\alpha\sin\alpha}{\Sigma}\,d\hat{x}\,d\hat{r} - \\ &\quad - \frac{4\,m\,a\,t\,\sinh^2\theta\cos\alpha}{\Sigma}\,d\phi\,d\hat{r} - \frac{4\,m\,a\,t\,\sinh^2\theta\sin\alpha}{\Sigma}\,d\phi\,d\hat{x} \\ \Phi &= 0 \\ B &= 0\,. \end{aligned} \tag{3.14}$$

Step 3: we apply the T-duality transformation eq. (3.2) to the previous solution (3.14), using the symmetry associated with translations along the direction $\hat{x}$. We obtain the following new metric

$$\begin{aligned} d\tilde{s}_5^2 &= -\frac{\Sigma}{\Delta}\,dt^2 + \Sigma\,d\theta^2 + \frac{\Delta + a^2\sinh^2\theta}{\Delta + a^2\,\sinh^2\theta + 2\,m\,t\,\cos^2\alpha}\,d\hat{r}^2 + \tilde{g}_{\phi\phi}\,d\phi^2 + \\ &\quad + \frac{\Sigma}{\Delta + a^2\sinh^2\theta + 2\,m\,t\,\cos^2\alpha}\,d\hat{x}^2 - \frac{4\,m\,a\,t\,\sinh^2\theta\,\cos\alpha}{\Delta + a^2\,\sinh^2\theta + 2\,m\,t\cos^2\alpha}\,d\hat{r}\,d\phi\,, \end{aligned} \tag{3.15}$$

where $\tilde{g}_{\phi\phi}$ is given by the expression

$$\tilde{g}_{\phi\phi} = \frac{\sinh^2(\theta)}{\Delta + a^2\,\sinh^2\theta + 2\,m\,t\,\cos^2\alpha}\left[\Delta\,\Sigma + 2\,m\,t\,\cos^2\alpha\,(t^2+a^2)\right]. \tag{3.16}$$

The dilaton and antisymmetric form are similarly given by

$$\tilde{B}_{\hat{x}\hat{r}} = -\frac{2\,m\,t\,\cos\alpha\,\sin\alpha}{\Delta + a^2\sinh^2\theta + 2\,m\,t\,\cos^2\alpha}\,, \tag{3.17}$$

$$\tilde{B}_{\hat{x}\hat{\phi}} = -\frac{2\,m\,a\,t\,\sin\alpha\,\sinh^2\theta}{\Delta + a^2\sinh^2\theta + 2\,m\,t\,\cos^2\alpha}\,, \tag{3.18}$$

$$\tilde{\Phi} = -\ln\frac{\left[\Delta + a^2\sinh^2\theta + 2\,m\,t\,\cos^2\alpha\right]}{\Sigma}\,. \tag{3.19}$$

[8] We could easily add n flat directions, but we concentrate on one for the sake of clarity.

Taking the limit $\alpha = 0$, one recover the solution (3.12). In the limit $\alpha = \frac{\pi}{2}$, instead, one finds essentially the T-dual solution discussed in section 3.1, with the addition of an extra flat dimension.

The charged string configuration coupled to the antisymmetric tensor field constructed above is completely time dependent and regular for $a^2 > m^2$ (that is, Δ is always positive). When $a^2 < m^2$, internal, stationary regions are present, separated by Cauchy horizons from the external time dependent regions. In this case, curvature singularities are possibly present when the dilaton diverges, inside the stationary region, at points satisfying the equation

$$\Delta + a^2 \sinh^2\theta + 2mt \cos^2\alpha = 0 \,. \tag{3.20}$$

For this configuration, the presence of additional non zero components for the field B, in comparison to the simpler four dimensional case discussed in section 3.1, allows to obtain non singular solutions with horizons and non trivial dilaton: it is enough to choose the rotation angle α in such a way that the following inequalities are satisfied

$$0 < \sin^4\alpha < \frac{a^2}{m^2} \,, \tag{3.21}$$

as an analysis of the equation (3.20) can easily show. The internal, regular stationary regions, however, contain again CTC's.

3.4 SKerr in heterotic string theory

In this final section, we generate new solutions using the method of twisting in heterotic string theory, as introduced by Hassan and Sen [17]. In that work, a class of symmetries of the solutions to heterotic string theory was discovered for configurations which are independent of d of the space-time directions, and for which the background gauge fields are neutral under p of the U(1) generators of the gauge group. The symmetry identified by these authors is $O(d)\times O(d+p)$ if all of the d directions are space-like, or $O(d-1,1)\times O(d+p-1,1)$, if the signature of the d-coordinates is lorentzian.

For our purposes, our interest in this symmetry is to generate new inequivalent time dependent solutions from known ones. Indeed, in [17], the authors applied this transformation to a static 6-brane in ten dimensions, to generate a new solution carrying electric and magnetic charges, antisymmetric tensor field charge and non trivial dilaton field. In [26], Sen derived in a similar way a rotating, charged black hole with nontrivial dilaton, magnetic fields and antisymmetric tensor field in heterotic string theory, starting from a rotating solution in the vacuum.

We are now interested in applying these transformations to the 5-dimensional Kerr S-string constructed adding a flat direction, x, to the 4-dimensional Kerr S-brane solution (our starting point is the first step in the previous subsection, that is, the solution in eq. (3.12)); our discussion will be based on [27]. Thus we start with the low-energy action for heterotic string theory in $D = 5$ dimensions, with the other $(10 - D)$ dimensions compactified. We do not include the massless compactification moduli in the effective action as well as higher derivative terms. Then, we are left with the 5-dimensional metric, the dilaton, the antisymmetric two form field and the Maxwell field, whose action is given by

$$S = -\int dx^5 \sqrt{-g_5}\, e^{-\Phi} \left[R + (\nabla\Phi)^2 - \frac{1}{12} H_3^2 - \frac{1}{8} F_2^2 \right] . \tag{3.22}$$

Here Φ is the dilaton field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the U(1) gauge field. The three form field H is given by $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho}$ + cyclic permutations - CS, where the U(1) Chern-Simons term is $CS = \frac{1}{4}(A_\mu F_{\nu\rho}$ + cyclic permutations).

We wish to apply the Hassan-Sen transformation to the solution in eq. (3.12), which is independent of the two spatial coordinates, (r, x) (that is, $d = 2$). Given a solution $(g_{\mu\nu}, \Phi, A_\mu, B_{\mu\nu})$, a transformed solution $(\tilde{g}_{\mu\nu}, \tilde{\Phi}, \tilde{A}_\mu, \tilde{B}_{\mu\nu})$ can be obtained in the following way.

Define the 11×11 matrix M as

$$M = \begin{pmatrix} (K^T-\eta)g^{-1}(K-\eta) & (K^T-\eta)g^{-1}(K+\eta) & -(K^T-\eta)g^{-1}A \\ (K^T+\eta)g^{-1}(K-\eta) & (K^T+\eta)g^{-1}(K+\eta) & -(K^T+\eta)g^{-1}A \\ -A^T\hat{g}^{-1}(K-\eta) & -A^T g^{-1}(K+\eta) & A^T g^{-1} A \end{pmatrix} , \tag{3.23}$$

where $K_{\mu\nu}$ is a 5×5 matrix defined in terms of the original solution,

$$K_{\mu\nu} = -B_{\mu\nu} - g_{\mu\nu} - \frac{1}{4} A_\mu A_\nu \, , \tag{3.24}$$

and η =diag$(1, 1, 1, 1, 1)$.[9]

Our original solution (3.12) has $A_\mu = 0$, and so we have $d = 2$ and $p = 1$. Thus the symmetry of the space of solutions is $O(2) \times O(3)$. The Hassan-Sen transformation then acts on the original solution, contained in the matrix M, to give a new solution:

$$\tilde{M} = \Omega M \Omega^T \tag{3.25}$$

$$\tilde{\Phi} = \Phi + \ln \sqrt{\frac{\det \tilde{g}}{\det g}} , \tag{3.26}$$

where $\Omega \in O(2) \times O(3)$ is given explicitly by

$$\Omega = \begin{pmatrix} I_3 & & & \\ & S & & \\ & & I_3 & \\ & & & R \end{pmatrix} . \tag{3.27}$$

Here, I_3 represents the identity, while $R \in O(3)$ and $S \in O(2)$. We can choose R to be of the form

$$R = \begin{pmatrix} \cos\alpha_2 & \sin\alpha_2 & 0 \\ -\sin\alpha_2 & \cos\alpha_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha_1 & 0 & \sin\alpha_1 \\ 0 & 1 & 0 \\ -\sin\alpha_1 & 0 & \cos\alpha_1 \end{pmatrix} , \tag{3.28}$$

[9] In the case in which one of the coordinates in d is time-like, $\eta = (-1, 1, \ldots, 1)$.

where α_1 parameterizes rotations which mix the r direction with the internal coordinate, and α_2 parameterizes rotations in the (r, x) space. The $O(2)$-transformations are rotations of the solution in the (r, x)-plane and we choose S to be the identity matrix.

With this choice for Ω, applied to the matrix M constructed in terms of the original solution, we can determine the new solution completely. The new metric is given by

$$\begin{aligned} d\tilde{s}_5^2 &= -\frac{\Sigma}{\Delta}\,dt^2 + \Sigma\,d\theta^2 + dx^2 + \frac{[\Sigma(\Delta + a^2\sinh^2\theta) + \sin^2\alpha_2\cos^2\alpha_1\,m^2\,t^2]}{[m\,t\,(1-\cos\alpha_1\cos\alpha_2) - \Sigma]^2}\,dr^2 + \\ &+ \tilde{g}_{\phi\phi}\,d\phi^2 + \frac{2\,m\,t\,a\sinh^2\theta[\sin^2\alpha_2\cos^2\alpha_1\,m\,t - (1+\cos\alpha_1\cos\alpha_2)\Sigma]}{[\Sigma - m\,t\,(1-\cos\alpha_1\cos\alpha_2)]^2}\,dr d\phi + \\ &+ \frac{2\,m\,t\,\sin\alpha_2\cos\alpha_1}{[m\,t\,(1-\cos\alpha_1\cos\alpha_2) - \Sigma]}\,dx dr + \frac{2\,m\,t\,\sin\alpha_2\cos\alpha_1\,\sinh^2\theta}{[m\,t\,(1-\cos\alpha_1\cos\alpha_2) - \Sigma]}\,dx d\phi\,, \end{aligned} \tag{3.29}$$

where

$$\begin{aligned} \tilde{g}_{\phi\phi} &= \frac{\sinh^2\theta}{\Sigma}\Bigg\{(t^2+a^2)^2 + a^2\sinh^2\theta\Delta + m^2t^2a^2\sinh^2\theta\,\times \\ &\times\left[\frac{2(1-\cos\alpha_1\cos\alpha_2)}{m\,t(1-\cos\alpha_1\cos\alpha_2) - \Sigma} - \frac{\sin^2\alpha_1\Sigma}{[m\,t(1-\cos\alpha_1\cos\alpha_2) - \Sigma]^2}\right]\Bigg\}\,. \end{aligned} \tag{3.30}$$

The dilaton and background fields are similarly given by

$$\begin{aligned} \tilde{\Phi} &= -\ln\left[\frac{\Sigma - mt(1-\cos\alpha_1\cos\alpha_2)}{\Sigma}\right], \\ \tilde{A}_\phi &= -\frac{2mta\sin\alpha_1\sinh^2\theta}{\Sigma - mt(1-\cos\alpha_1\cos\alpha_2)}\,, \\ \tilde{A}_r &= -\frac{2mt\sin\alpha_1}{\Sigma - mt(1-\cos\alpha_1\cos\alpha_2)}\,, \\ \tilde{B}_{r\phi} &= \frac{mta(1-\cos\alpha_1\cos\alpha_2)\sinh^2\theta}{\Sigma - mt(1-\cos\alpha_1\cos\alpha_2)}\,, \\ \tilde{B}_{rx} &= \frac{mt\sin\alpha_2\cos\alpha_1}{\Sigma - mt(1-\cos\alpha_1\cos\alpha_2)} \\ \tilde{B}_{x\phi} &= \frac{-mta\sin\alpha_2\cos\alpha_1\sinh^2\theta}{\Sigma - mt(1-\cos\alpha_1\cos\alpha_2)}\,, \end{aligned} \tag{3.31}$$

while the other components of $\tilde{A}_\mu$ and $\tilde{B}_{\mu\nu}$ vanish.

This string-like, time dependent solution has a global structure which resembles that of the charged S-string of last section. We have a completely time-dependent, regular, causally well defined configuration whenever $a^2 > m^2$. In the opposite case, there could be closed timelike curves in the internal region and naked singularities situated where the dilaton, the $\tilde{A}$ and $\tilde{B}$ fields are singular. This happens if the following equation has real roots

$$\Sigma - m\,t\,(1-\cos\alpha_1\cos\alpha_2) = 0\,. \tag{3.32}$$

Also in this case, for $a^2 < m^2$, one can judiciously choose the angles α_1 and α_2, in such a way that the metric is singularity free, although the internal stationary regions contain CTC's.

Let us end noticing that the solution we obtained in this section *via* a Hassan-Sen transformation, is actually connected to the previous solutions for certain choices of the angles α_i. For $\alpha_1 = \alpha_2 = 0$, one naturally finds the vacuum solution of eq. (3.12). More interestingly, consider the case $\alpha_1 = 0$, and $\alpha_2 = 2\alpha$. One can explicitly check that in this way the resulting solution reduces to the charged solution we presented in the third step of section 3.3, once we perform on the latter a new coordinate rotation (3.13), with the same angle α.

4. Conclusions

In the present letter we have identified several classes of exact time-dependent solutions to the low-energy supergravity equations in four and higher dimensions. Unlike many of the S-brane solutions discussed earlier in the literature, some of the new solutions we obtain are particularly interesting because they are nonsingular, inasmuch as they have neither horizons, curvature singularities or closed timelike curves.

We discuss the simplest such solution — the Kerr S-brane — in some detail. This is obtained from the Kerr solution by making complex substitutions of coordinates and parameters in precisely the same manner as the more symmetric S-brane solutions have been constructed from the Schwarzschild spacetime. The solution contains two parameters, a and m, and reduces to the earlier S-brane configurations as $a \to 0$. We show that these solutions are nonsingular and causally well behaved if they satisfy $a^2 > m^2$. When $0 < a^2 < m^2$, although the solution is still free of curvature singularities, it is characterized by internal stationary regions that contain closed time-like curves, which make them problematic from the point of view of causality. It might be interesting to study issues such as stability and particle production in these simple non-singular, time dependent backgrounds.

We then generalize the simplest solution to include more dimensions, and non trivial configurations for the typical fields of low-energy supergravity, applying various solution generating techniques. We concentrate on S-string backgrounds in five dimensions, that contain a scalar field, and antisymmetric one- and two-form fields. Also in these richer situations, we are able to identify time dependent solutions that are completely regular in the entire space-time. Simple extensions of these five dimensional solutions to higher dimensions can be obtained by taking a product with additional flat directions. Other generalizations can be achieved by performing different kinds of duality transformations.

It would be interesting to understand the connection between the tachyon field dynamics in string theory and the regular time dependent solutions we have presented here. Also, it is worth to investigate the possibility to construct realistic cosmological models based on these geometries.

Note added: while this letter was in the last stages of preparation, we received the preprint in reference [13], which contains substantial overlap with the first part of the present work.

JHEP04(2004)038

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JHEP04(2004)038

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